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The 24 in. Schmidt satellite cameras, and their use in geodetic and geophysical investigations

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[Plates 3 and 4]

Although Britain has not established a network of optical stations for tracking artificial Earth satellites, very many optical observations have been made both by an excellent group of amateurs who make visual observations to an accuracy of 1′ to 2′ of arc and 0·1 s of time and by a number of kinetheodolite stations. Precision measurements of satellite positions are also made using two cameras which have been designed and built in Britain. These cameras are comparable with the Baker-Nunn cameras used by the Smithsonian Astrophysical Observatory of America in their world-wide network of optical tracking stations, and like the Baker-Nunn cameras they have been specially designed for observing satellites. The British cameras, however, unlike the Baker-Nunn cameras, do not track the satellite but record its trail from a bearing and elevation fixed in relation to the Earth. Consequently, they are not as sensitive as the Baker-Nunn cameras but errors which might arise from the tracking system have been eliminated.

The sensitivity of a fixed camera depends not only upon the sensitivity of the photographic emulsion but also upon the aperture and focal length of the optical system and its optical quality. The limiting magnitude of a system is given by

$$\frac{E}{k}\left(\frac{d}{D}\right)^2\frac{1}{t},$$

where E is the sensitivity of the photographic emulsion, d, the diameter of the image, D, the diameter of the camera aperture, K the transmission coefficient of the system and t the exposure time. For a fixed camera the exposure time, t, can be expressed as $d/\omega F$, where ω is the angular velocity of the satellite in radians per second and F is the focal length of the optical system. For a tracking camera, t is given by $d/|V-V_0|$, where $|V-V_0|$ is the difference between the velocity of the satellite and the tracking velocity of the camera. The sensitivity of a tracking camera with a given aperture, therefore, increases as its tracking velocity approaches that of the satellite and, with an accurately predicted velocity of the satellite, a considerable increase in the camera sensitivity can be obtained. However, the sensitivity of a fixed camera of the same aperture can be increased only by reducing the focal length. As measuring accuracy is determined by the scale of the image on the photographic plate, the focal length must not be too short, otherwise accuracy will suffer.

Most satellites have magnitude fainter than +5 and a wide range of angular velocities varying from less than 0·1° per second to more than 1° per second. Earlier work at the Royal Radar Establishment (Hewitt 1960) showed that cameras of 90 cm focal length and 14 cm aperture are incapable of recording artificial Earth satellites moving with an angular velocity of 1° per second if their brightness is less than +4 magnitude. Therefore, if most satellites are to be recorded, it is essential that the optical system should have a very wide aperture which, combined with the increased sensitivity obtained by reducing the focal length of the optical system in a fixed camera, leads to the choice of a system of small *f* number.

The optical system of the R.R.E. Satellite camera (Hewitt 1965) is a flat field *f*/1 Schmidt system with an aperture of 60 cm. The camera (figure 1, plate 3) was designed and built at the Royal Radar Establishment, the optical system being produced by Grubb Parsons Ltd of Newcastle upon Tyne. It has a field of 10° diameter and at the centre of the field the images of +8 magnitude stars recorded on fast panchromatic emulsion are 42 μm diameter.

The methods of measuring the precise positions of satellites use the star background as a reference system and the time at which the star exposures are made must therefore be recorded. For this purpose, a large five-bladed iris shutter of 65 cm diameter is mounted in front of the corrector plate of the optical system. The time at which this capping shutter opens and closes can be recorded and the duration of the star exposures can be as short as $\frac{1}{4}$ s. To avoid vibration this shutter is mounted on a separate turntable, concentric with the turntable carrying the optical system.

Such a large shutter cannot be operated at a sufficiently high speed to produce short breaks, approximately 100 μm long, which are the datum points on the satellite trail. Consequently, a second shutter has been provided. This is a rotating sector shutter mounted near the field lens and passing through a gap between the field lens and the photographic plate. The time is recorded every revolution of the sector shutter where it passes the slit in a timing unit, consisting of a phototransistor and lamp, mounted near the field lens. Four fiducial marks are recorded on the photographic plate. One of the fiducial systems is mounted on the centre of rotation of the sector shutter and another lies on the line joining the centre of rotation of the shutter and the slit of the timing unit. The positions at which the time is recorded are, therefore, defined on the photographic plate. From the measurements of the four fiducial marks and the recorded times from each revolution of the sector shutter, the precise time of each break in the satellite trail can be calculated. The breaks in the satellite trail and the recorded times are correlated by means of a code which is produced by closing the capping shutter for one or more revolutions of the sector shutter.

The time is recorded by means of a quartz clock driven by a 1 Mc/s oscillator and the time is displayed to four decimals of a second, by means of digitrons. The display of the decimal seconds changes only when an incoming signal is received. A camera is used to record the time displayed on the receipt of the signals.

The accuracy of the camera has been investigated by measuring a group of star images, recorded at intervals across the field of the camera. The results of these investigations have shown that the positions of stars can be determined with a standard deviation of $\pm 0{\cdot}9''$. A similar series of measurements of breaks in star trails, simulating the breaks in satellite trails, gave a standard deviation of $\pm 1{\cdot}1''$.

The rotating sector shutter is driven by a velodyne motor system through a specially designed fibre-glass belt and there are slight variations in the period of revolution of the shutter. This jitter and the mechanical tolerances in the construction of the shutter introduce errors into the calculated time of the break in the satellite trail. These errors are in the order of $\pm 0{\cdot}1\,\%$ of the period of revolution of the shutter. Thus with the shutter rotating at the maximum rate of 5 rev/s the timing accuracy is $\pm 0{\cdot}0002$ s.

The sensitivity of the camera has also been determined by sweeping the camera at an angular velocity equivalent to 1°/s across a star field. With very fast emulsions the limiting magnitude for satellites moving with this angular velocity is +7 to +7·5.

Table 1. Performance of R.R.E. satellite camera

	accuracy of measurement
star images	0·9″
gaps in trails	1·1″
timing error	±0·1 % of period of revolution of shutter
	sensitivity
satellite at 1°/s angular velocity	+7 limiting magnitude

The field flattener in the optical system introduces considerable distortion in the image field. At a radius of 5 cm from the optical axis the image is displaced by approximately 300 μm. With this large distortion the methods of calculating the calibration parameters and the distortion coefficients, described by Brown (1957) and others, cannot be used without modification. In the calibration of a camera by the photogrammetric method the relation between the object space and the image space is given by

$$x-x_p = c\left[\frac{AX+BY+CZ}{DX+EY+FZ}\right] = \frac{cm}{q} \quad \text{and} \quad y-y_p = c\left[\frac{A'X+B'Y+C'Z}{DX+EY+FZ}\right] = \frac{cn}{q},$$

where x, y are the coordinates of the measured point, x_p, y_p the coordinates of the principal point and c is the principal distance. X, Y, Z are the coordinates of the front nodal point of the camera and A, B, C, ... are functions of three angles relating the coordinate axes of the image space to those of the object space. Radial distortion in an optical system can be expressed in terms of the odd powers of the distance between the image point and the optical axis. Converting this radial distance, r, into distances along the x and y axes and introducing the distortion terms into the above expressions gives

$$(x-x_p)\left(1+\sum_{i=1}^{n} h_i r^{2i}\right) = \frac{cm}{q} \quad \text{and} \quad (y-y_p)\left(1+\sum_{i=1}^{n} h_i r^{2i}\right) = \frac{cn}{q},$$

where the h_i are the distortion coefficients. The computation of the calibration parameters is carried out by introducing approximate values of these parameters. With these parameters, the residuals are

$$v_i = (x_i-x_p)\left(1+\sum_{i=1}^{n} h_i r^{2i}\right) - \frac{cm}{q} = F_i,$$

$$v_i' = (y_i-y_p)\left(1+\sum_{i=1}^{n} h_i r^{2i}\right) - \frac{cn}{q} = G_i.$$

These expressions are linearized about the measured plate coordinates x^0, y^0 and the approximate parameters in a Taylor expansion giving

$$v_i = F_i^0 + \frac{\delta F_i^0}{\delta x_p}\Delta x_p + \frac{\delta F_i^0}{\delta y_p}\Delta y_p \ldots \frac{\delta F_i^0}{\delta h_n}\Delta h_n$$

$$v_i' = G_i^0 + \frac{\delta G_i^0}{\delta x_p}\Delta x_p + \frac{\delta G_i^0}{\delta y_p}\Delta y_p \ldots \frac{\delta G_i^0}{\delta h_n}\Delta h_n,$$

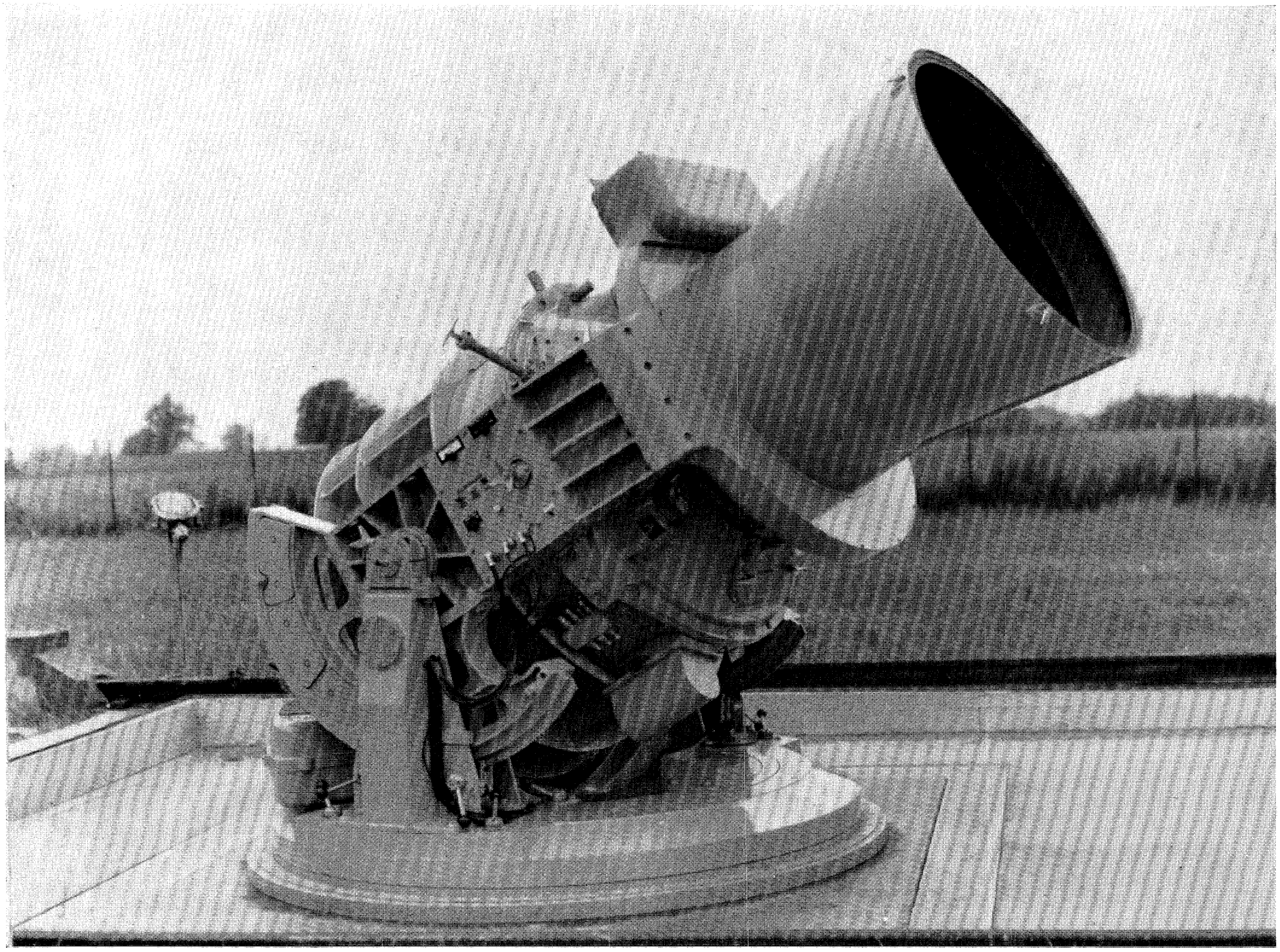

FIGURE 1. The R.R.E. satellite camera.

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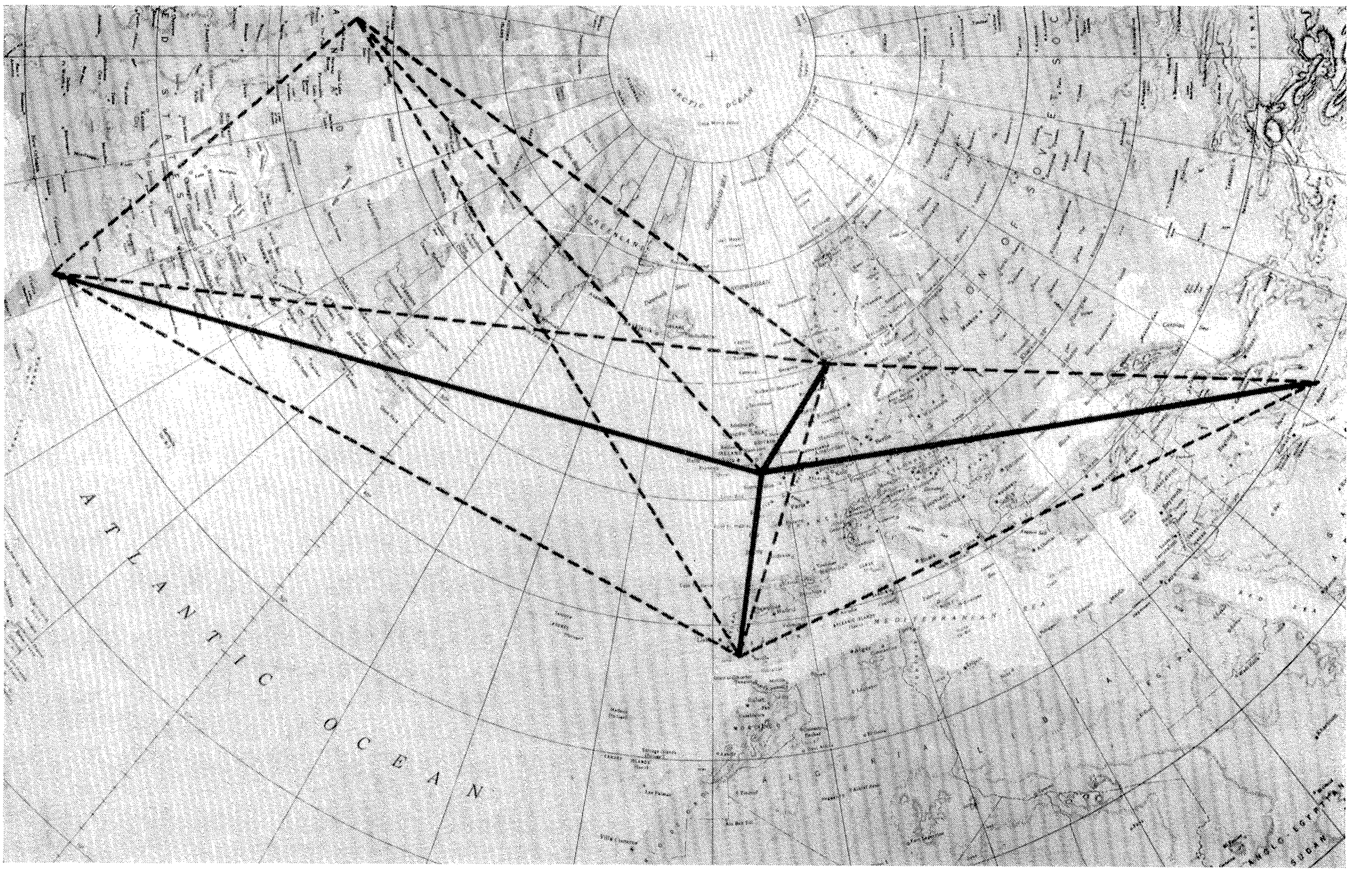

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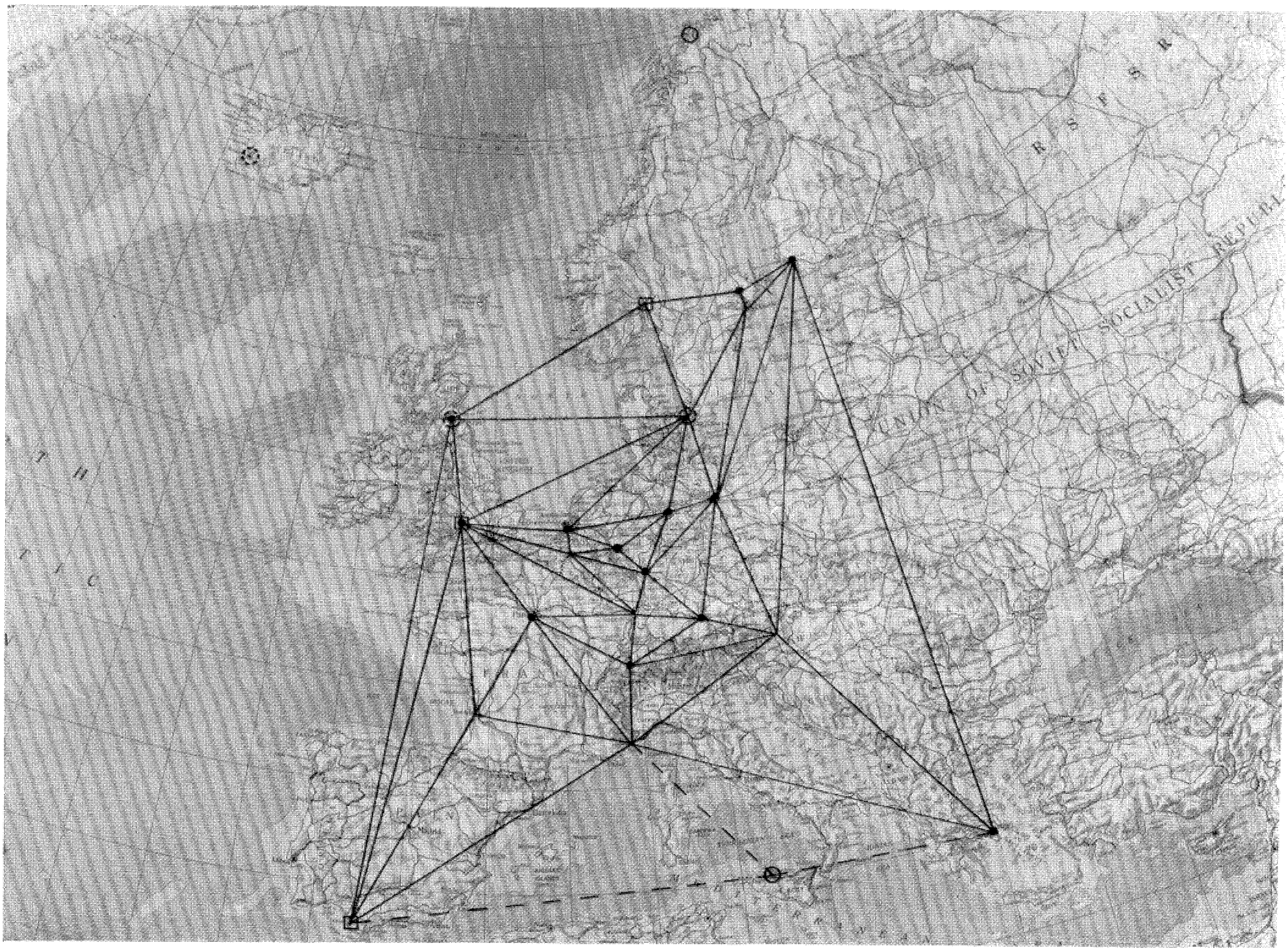

FIGURES 2 AND 3. (For descriptions see opposite.)

and the solution is obtained by iteration using a least squares process to minimize the sum of the squares of the residuals.

Currie (1964) at the Royal Radar Establishment, Malvern, has shown that if $|2h_1c^2| \geqslant 1$ the iteration will not converge and terms in h must be included in the expansions of

$$\frac{\delta F_i^0}{\delta x_p}, \quad \frac{\delta F_i^0}{\delta y_p}, \quad \frac{\delta G_i^0}{\delta x_p} \quad \text{and} \quad \frac{\delta G_i^0}{\delta y_p}.$$

In the R.R.E. satellite camera, $c \simeq 0{\cdot}6$ m and $h_1 \simeq 1{\cdot}8\ \mathrm{m}^{-2}$ and $|2h_1c^2|$ is approximately equal to 1·3. It has, therefore, been necessary to include terms in h_1 in the expansions of the four terms in order to calibrate the camera and determine the distortion coefficients. When the distortion is removed, the root mean square residual for a star image is 4·4 μm, equivalent to 1·5″. With approximately 100 measured star images the computed direction of the optical axis has a root mean square error of 1·05 μm or 0·3″.

For the successful operation of a fixed camera system with a 10° field, accurate predictions of the positions of the satellite at known times must be available. If the predictions are accurate to $\frac{1}{2}^\circ$ and of the order of 6 s of time, the camera can be set to the predicted direction and the exposure made at the predicted time. Many satellites, particularly those at high altitudes, can be predicted to this or better accuracy. Satellites with low perigee and also balloon type satellites, such as Echo, which are used for geophysical investigations, are often predicted with considerably less accuracy, the time being the most serious source of error. If the error in the predicted time is less than 2 or 3 min, an optical tracking sight is used to control the operation of the camera. This sight has bearing and elevation axes and a third axis normal to the elevation axis. The bearing and elevation axes can be adjusted so that the optical axis of the sight rotates about the third axis in the approximate plane of the satellite orbit. The sighting telescope is turned about the third axis by a variable speed motor set to the predicted velocity of the satellite and differential corrections to the motion can be made by hand. Contacts, coupled to the telescope drive, operate the camera through a control unit. The contacts can be set so that the camera shutter opens as the satellite is tracked into the field of the camera and other contacts on the sight automatically control the precalibration and post-calibration star exposures.

Two R.R.E. satellite cameras have been made and they were installed by late 1962, but no routine observations were made for two years. In February 1965 a programme of routine observations was initiated in cooperation with the Smithsonian Astrophysical Observatory. Many successful observations have been made of the high altitude satellites, Midas 4 and 1963–30D, and of other satellites, simultaneous observations having been obtained between the British camera at Malvern and the Baker-Nunn stations in Florida, Spain, Norway and Iran (figure 2, plate 4). The measurement of the photographic plates

DESCRIPTION OF PLATE 4

FIGURE 2. The participation of the R.R.E. satellite camera at Malvern, England, in the S.A.O. simultaneous observation programme. Simultaneous observations have been made between Malvern and the stations indicated by the full lines.

FIGURE 3. The Western European Satellite Triangulation Project and the association of the R.R.E. cameras with the various geodetic projects (see text).

is being undertaken by the S.A.O. The data have not yet been fully reduced but preliminary results show that satellite positions determined from the observations made by the camera at Malvern fit the mean orbit derived from the Baker-Nunn network with a root mean square error of 2″.

When the cameras were installed in 1962 attempts were made to observe the Anna 1B flashing satellite but without success. In November 1965, the National Aeronautics and Space Administration launched the first satellite, Geos A, in their geodetic programme. The Malvern station is an international participant in this programme. Regular observations of this satellite are being made also as part of the S.A.O. simultaneous observation programme and over 100 successful observations have been obtained.

Late in 1964 the International Association of Geodesy set up a Western European Subcommission to establish a European Satellite Triangulation Network. Most of the Western European countries are cooperating in this programme, simultaneously observing the satellites Echo 1, Echo 2 and Pageos. The observing schedule started on 1 August 1966 and approximately 26 stations are taking part. These stations are distributed over most of West Europe (as shown in figure 3, plate 4), the majority being in the central area. Those stations marked in the diagram with a large dot are also participants in the N.A.S.A. Geodetic Programme. The Baker-Nunn stations at San Fernando (Spain) and Oslo, and the stations at Malvern and Athens, associated with the S.A.O. simultaneous programme, are indicated by squares.

The second British camera has been moved recently to Earlyburn, the Satellite Tracking Station of the Royal Observatory, Edinburgh. This has increased the baseline between the two British cameras and has also enabled Britain to make a tie with the observations of the United States Coast and Geodetic Survey whose stations are shown by circles. Thus, it will be seen that the British cameras are a very strong link between these various geodetic programmes.

Not only are the cameras being used in these international geodetic projects but also satellites are being observed for the geophysical investigations whch are being made in Britain. Until recently only a very few observations were attempted but a more active programme has now been started.

Table 2. Analysis of observations: satellite camera at Malvern, U.K.

total nights, 500; nights operational, 190

			operational nights plates	
project	period	predicted observations	exposed	successful
S.A.O.	1. ii. 65 to 1. viii. 66	620	390	120
Geos	30. xi. 65 to 1. x. 66	230	180	100
W. European triangulation	1. viii. 66 to 1. x. 66	100	60	50
Geophysical Research (King-Hele)	1. viii. 66 to 1. x. 66	120	30	10
total	—	1070	660	280

The camera at Malvern has been making observations on all possible nights during the last 18 months. An analysis of the observations, given in table 2, shows that out of a total of 500 nights only 190 were suitable for observing. 1070 observations were predicted for the 190 nights, an average of about 6 per night. Only two-thirds of these observations were attempted and only 26% were successful. Based on the total number of nights, the success rate is approximately 10%.

It is interesting to compare the successful results which have been obtained on each project. Observations for the Western European Triangulation and on Geos are the most successful. Geos is accurately predicted and failures are mostly due to poor visibility or light cloud. Although the Echo satellites which are observed for the Western European programme are not so well predicted, they can usually be acquired by the optical sight and tracked through the field of the camera. An exposure, when made, is rarely unsuccessful. The predictions for the S.A.O. simultaneous observation programme are also accurate, but the satellites are dim and at long ranges. The images are, therefore, faint and the plates are frequently rejected because the track is not sufficiently dense. The satellites used for geophysical investigations are not very accurately predicted and, because their shape is irregular, they vary in brightness. They are, therefore, not always easy to see in the tracking sight and even when an exposure is made, the image can be of very uneven density and not suitable for measurement.

The routine observations have been carried out by a team of eight people who have also been responsible for the design and development of the equipment. Maximum emphasis has been given to the observation of satellites. Some of the photographic plates are measured and the data reduced at the Royal Radar Establishment, Malvern. However, it would be impossible for the small team to reduce all the plates in a reasonable time and the assistance of the Smithsonian Astrophysical Observatory in plate measurement is gratefully acknowledged.

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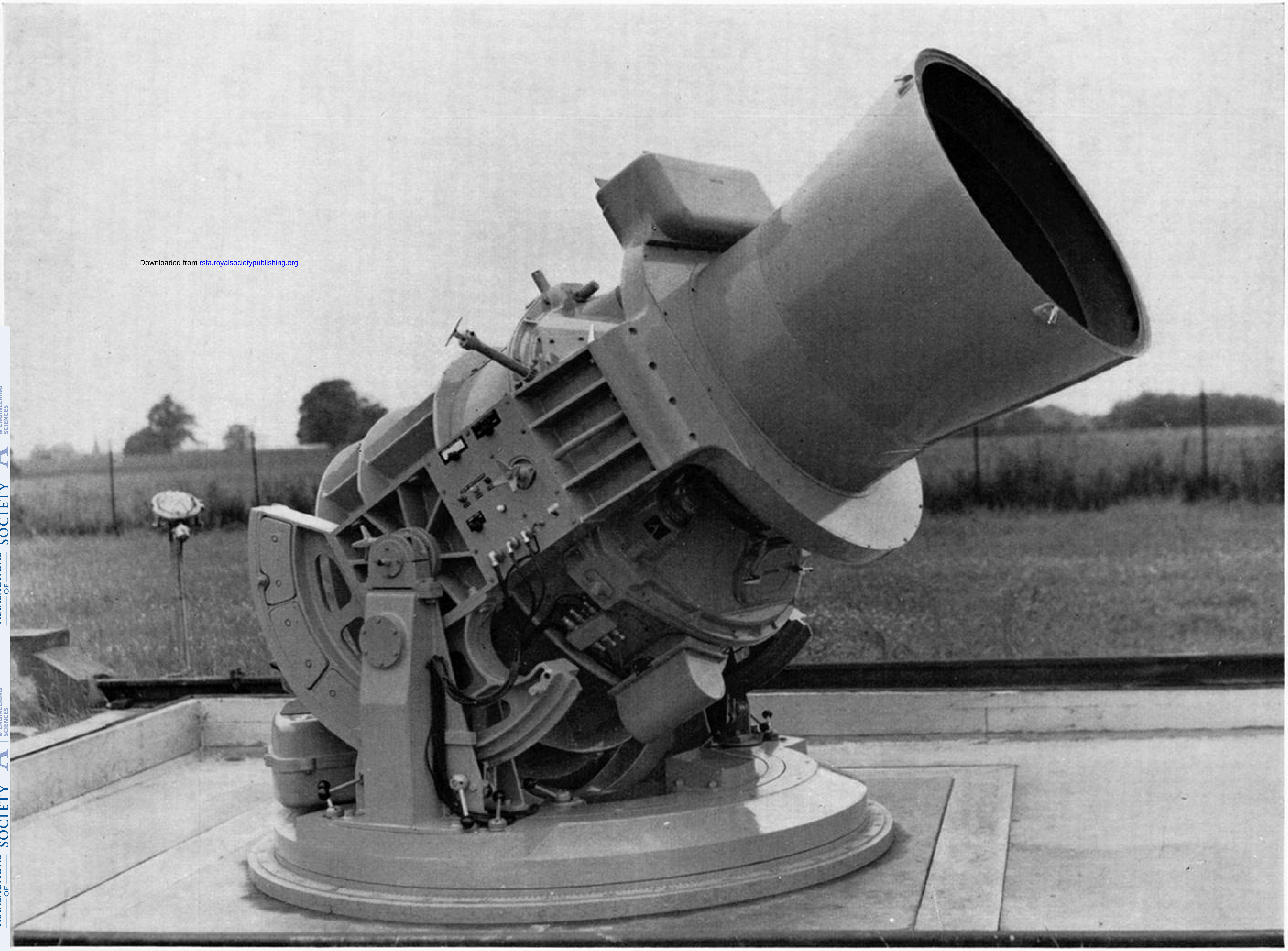

FIGURE 1. The R.R.E. satellite camera.

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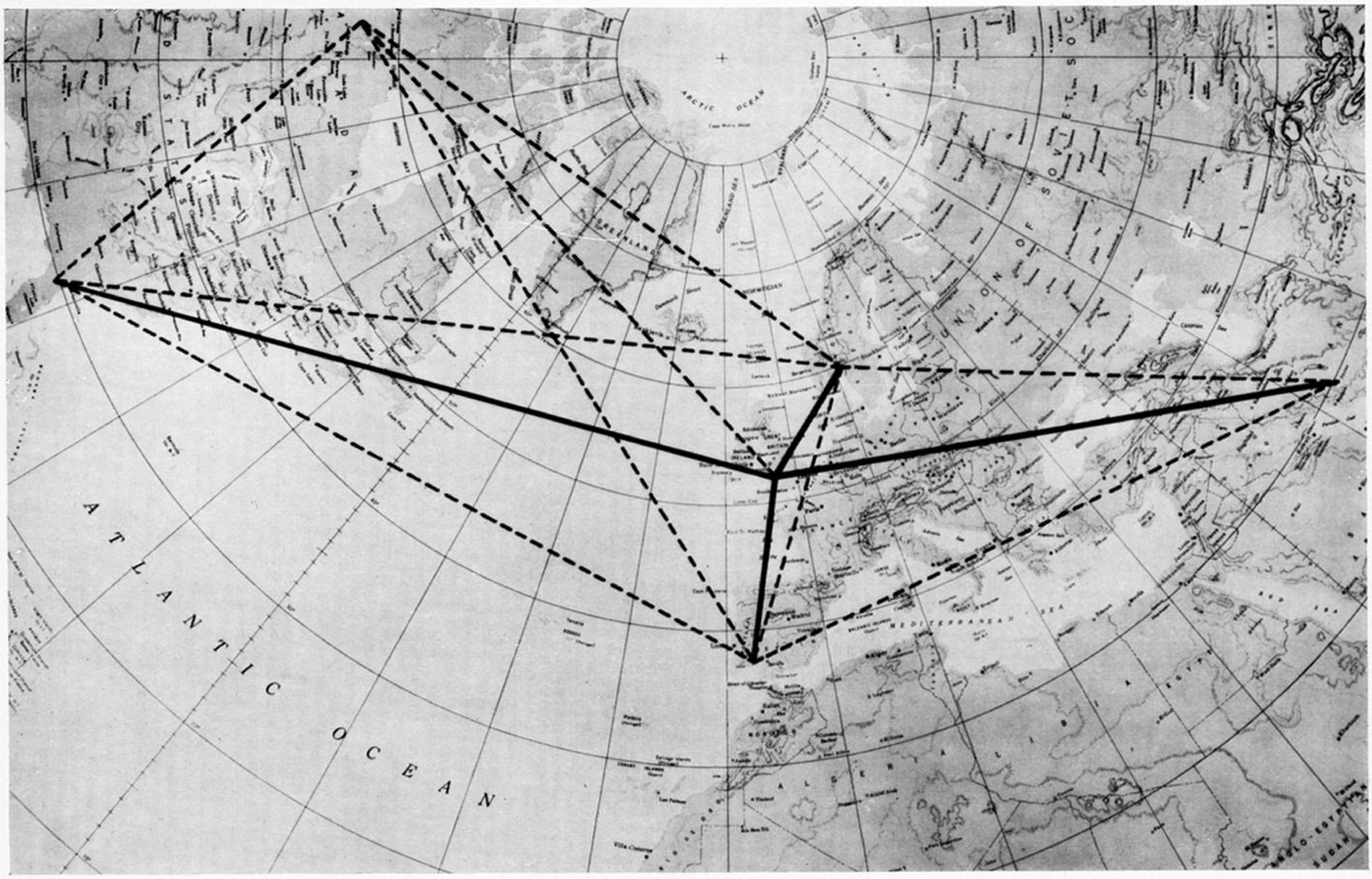

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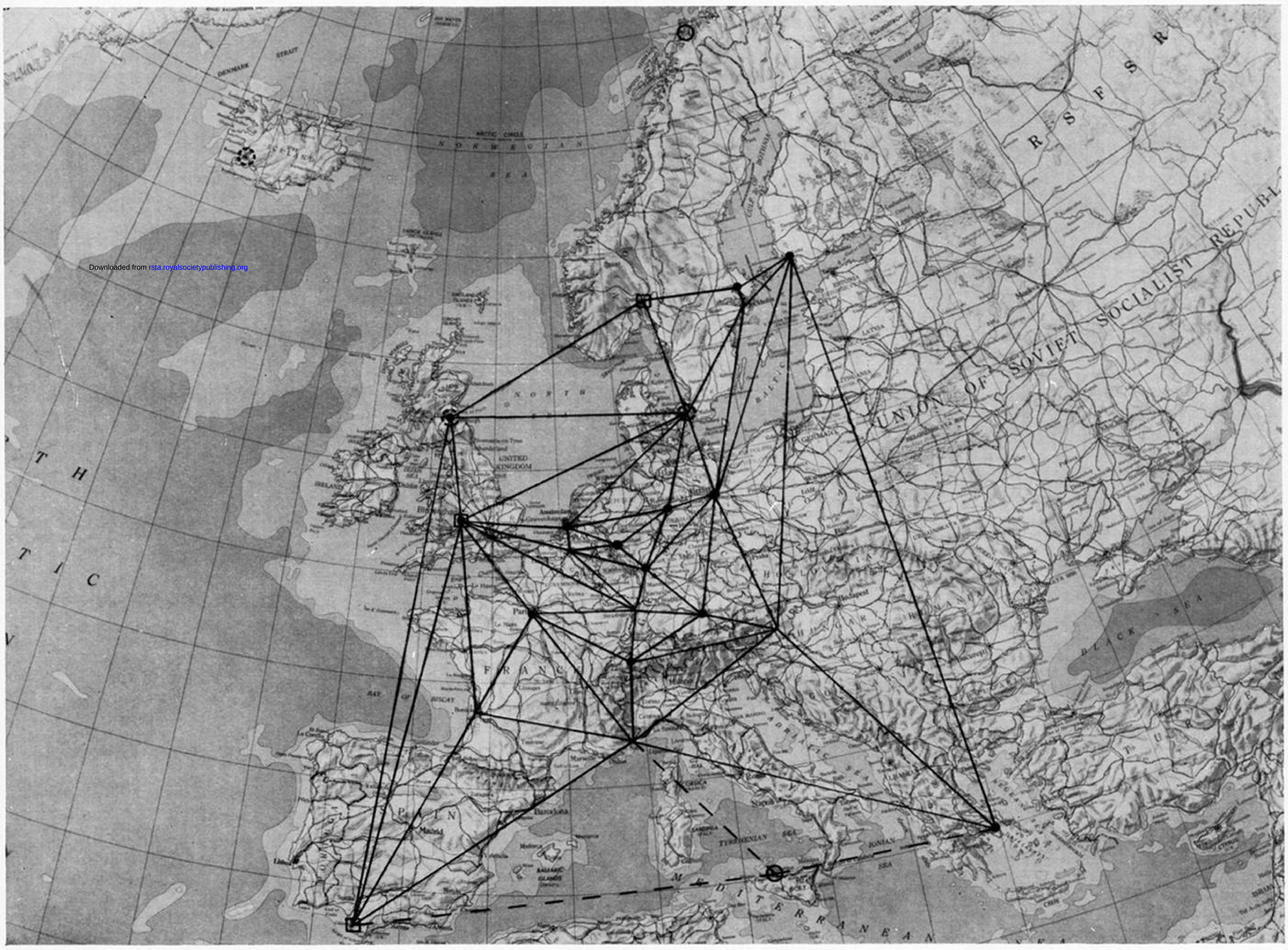

FIGURES 2 AND 3. (For descriptions see opposite.)